QUICK RECAPE

- **Charge :** Electric charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.
- **Quantization :** Charge is always in the form of an integral multiple of electronic charge and never its fraction.
  $q = \pm ne$
  where n is an integer and $e = 1.6 \times 10^{-19}$ C.
- Millikan's oil drop experiment showed the discrete nature of charge. Charge cannot be fractional multiple of e.
- **Conservation of charge :** Net charge of an isolated physical system always remains constant. Charge can neither be created nor destroyed. It can be transferred from one body to another.
- Electric charge is additive, *i.e.*, total charge is the algebraic sum of the individual charges.
- Electric charge is invariant as it does not depend upon the motion of the charged body or the observer.
- **Coulomb's inverse square law :** It states that the electrostatic force of attraction or repulsion acting between two stationary point charges is given by

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

where F denotes the force between two charges q_1 and q_2 separated by a distance r in free space. ε_0 is a constant known as permittivity of free space. Free space is vacuum and may be taken to be air practically.

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \frac{\text{N m}^2}{\text{C}^2}$$

- If free space is replaced by a medium, then ε_0 is replaced by $(\varepsilon_0 K)$ or $(\varepsilon_0\varepsilon_r)$ where K is known as dielectric constant or relative permittivity.

$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$$

$$K = \frac{\varepsilon}{\varepsilon_0} \text{ or } \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$K = 1$ for vacuum (or air) and $K = \infty$ for conductor/metal.

$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$ and its dimensional formula is $[\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$.

Vector form of the law (q_1 and q_2 are like charges)

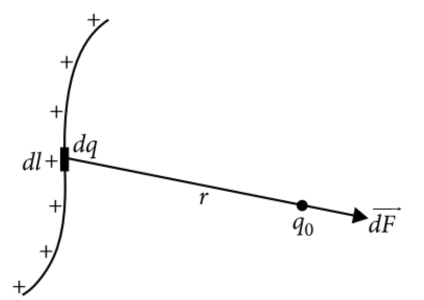
(i) $$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^3}\vec{r}_{21}$$

(ii) $$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r_{12}^3}\vec{r}_{12}$$

- **Electrostatic force due to continuous charge distribution**

The region in which charges are closely spaced is said to have continuous distribution of charge. It is of three types :

- **Linear charge distribution**



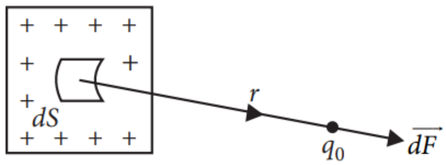
$dq = \lambda\, dl$

where, λ = linear charge density

$$\vec{dF} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0(dq)}{|r|^2}\hat{r} \Rightarrow \vec{dF} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0(\lambda dl)}{|r|^2}\hat{r}$$

Net force on charge q_0, $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_l \frac{\lambda dl}{|r|^2}\hat{r}$

- **Surface charge distribution**

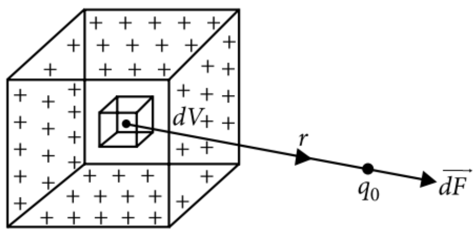


$dq = \sigma\, dS$

where, σ = surface charge density

Net force on charge q_0, $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_S \frac{\sigma dS}{|r|^2}\hat{r}$

- **Volume charge distribution**



$dq = \rho\, dV$

where, ρ = volume charge density

Net force on charge q_0, $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho dV}{|r|^2}\hat{r}$

Electric field intensity : The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge. It is expressed as

$$\vec{E} = \lim_{q_0\to 0} \frac{\vec{F}}{q_0}$$

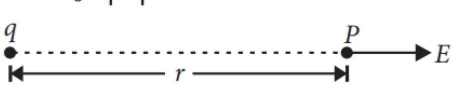
Here, $q_0 \to 0$, *i.e.*, the test charge q_0 must be small, so that it does not produce its own electric field.

SI unit of electric field intensity (E) is N/C and it is a vector quantity.

- **Electric field intensity due to a point charge**

Electric field intensity at P is,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|r|^2}\hat{r}$$



The magnitude of the electric field at a point P is given by

$$|E| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

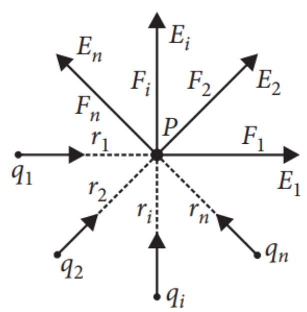
If $q > 0$, *i.e.*, positive charge, then E is directed away from source charge. On the other hand if $q < 0$, *i.e.*, negative charge, then E is directed towards the source charge.

$$E \propto \frac{1}{r^2}$$

- **Electric field due to a system of charges**

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{|r|^2}\hat{r}_i$$



A system of charges

Electric dipole : Two equal and opposite charges (q) each, separated by a small distance ($2l$) constitute an electric dipole. Many of the atoms/molecules are dipoles.

(i) Electric dipole moment, $\vec{p} = 2q\vec{l}$.

(ii) Dipole moment is a vector quantity and is directed from negative to positive charge.

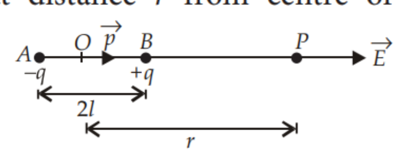
(iii) Unit of dipole moment is coulomb metre (Cm).

(iv) Dimension of dipole moment = [ATL]

- **Intensity of electric field due to a dipole**

– Along axis at distance r from centre of dipole

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

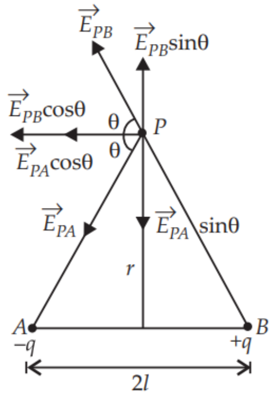


Direction of E is along the direction of dipole moment.

– Along equator of dipole at distance r from centre

$$\vec{E} = \frac{-1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$$

Direction of E is antiparallel to direction of p.



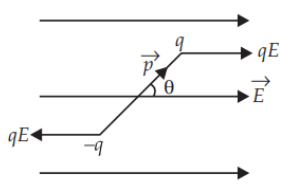
– At any point along direction θ

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

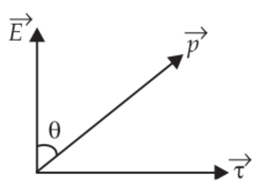
The direction of E makes an angle β with the line joining the point with centre of dipole where $\tan\beta = \frac{1}{2}\tan\theta$.

Electric dipole in a uniform electric field

- The resultant electric force on dipole = $qE - qE = 0$



- Two forces [qE and $(-qE)$] equal and opposite, separated by a distance constitute a couple (torque).
- Torque on a dipole $= pE\sin\theta$ numerically. Vectorially, Torque $(\vec{\tau}) = \vec{p}\times\vec{E}$



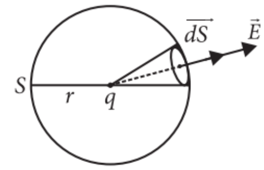
- The direction of τ is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$.
- The torque tends to align the dipole in the direction of field.
- Torque is maximum when θ = 90° *i.e.*, dipole is perpendicular to E.

  ∴ Maximum torque = pE.

  When θ = 0° or 180° then $\tau_{min} = 0$.
- When dipole is parallel to electric field, it is in stable equilibrium. When it is antiparallel to electric field, it is in unstable equilibrium.

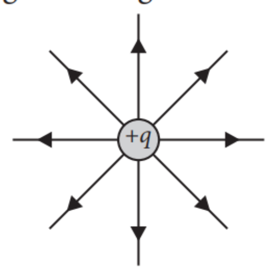
Gauss's law

Gauss's law : For a closed surface enclosing a net charge q, the net electric flux ϕ emerging out is given by $\phi = \oint_S \vec{E}\cdot\vec{dS} = \frac{q}{\varepsilon_0}$

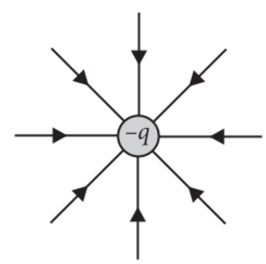


- If a dipole is enclosed by a closed surface, flux ϕ is equal to zero.

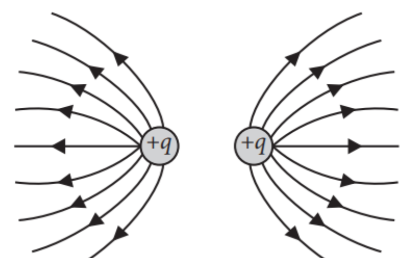
  Here the algebraic sum of charges $(+q - q = 0)$ is zero.
- The electric field lines due to positive and negative charges and their combinations are :



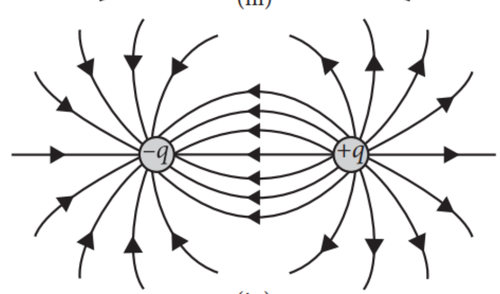
(i)



(ii)



(iii)



(iv)

- Flux from a cube

  (i) If q is at the centre of cube, total flux $(\phi) = \frac{q}{\varepsilon_0}$

  (ii) From each face of cube, flux $= \frac{q}{6\varepsilon_0}$
- Electric field due to a thin, infinitely long straight wire of uniform linear charge density λ,

$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}$, where r is the perpendicular distance of the observation point from the wire.

- Electric field due to uniformly charged thin spherical shell of uniform surface charge density σ and radius R at a point distant r from the centre of the shell is given as follows :
  At a point outside the shell *i.e.*, $r > R$, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
  At a point on the shell *i.e.*, $r = R$, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$
  At a point inside the shell *i.e.*, $r < R$, $\vec{E} = 0$
  Here, $q = 4\pi R^2\sigma$
- Electric field due to a thin non conducting infinite sheet of charge with uniform surface charge density σ is $\vec{E} = \frac{\sigma}{2\varepsilon_0}$
- Electric field between two infinite thin plane parallel sheets of uniform surface charge density σ and – σ is $\vec{E} = \sigma/\varepsilon_0$.
- Gaussian surface
  – For a sphere or spherical shell : A concentric sphere.
  – For cylinder or an infinite rod : A coaxial cylinder.
  – For a plate : A cube or a cuboid.

Coulomb's Law

VSA (1 mark)

1. Two equal balls having equal positive charge 'q' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two?

(AI 2014)

SA I (2 marks)

2. Plot a graph showing the variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$, where r is the distance between the two charges of each pair of charges : (1μC, 2μC) and (2μC, – 3μC), interpret the graphs obtained.

(AI 2011)

3. An infinite number of charges, each of q coulomb, are placed along x-axis at x = 1m, 3m, 9m and so on. Calculate the electric field at the point $x = 0$, due to these charges if all the charges are of the same sign.

(Delhi 2009)

Electric Field

SA I (2 marks)

4. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.

(Delhi 2007)

LA (5 marks)

5. Consider a system of n charges q_1, q_2 .... q_n with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, ... \vec{r}_n$ relative to some origin 'O'. Deduce the expression for the net electric field $\vec{E}$ at a point P with position vector $\vec{r}_p$ due to this system of charges.

(3/5, Foreign 2015)

Electric Field Lines

VSA (1 mark)

6. Why do the electrostatic field lines not form closed loops? *(AI 2014, AI 2012C)*

7. Why do the electric field lines never cross each other? *(AI 2014)*

SA I (2 marks)

8. The electric field $\vec{E}$ due to a point charge at any point near it is defined as $\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$, where q is the test charge and $\vec{F}$ is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression? Draw the electric field lines of a point charge Q when (i) $Q > 0$ and (ii) $Q < 0$. *(Delhi 2007)*

SA II (3 marks)

9. A point charge $(+Q)$ is kept in the vicinity of an uncharged conducting plate. Sketch the electric field lines between the charge and the plate. *(1/3, Foreign 2014)*

VSA (1 mark)

10. Write an expression for the flux $\Delta\phi$, of the electric field $\vec{E}$ through an area element $\Delta\vec{S}$. *(Delhi 2010C)*

SA I (2 marks)

11. (i) Define the term 'electric flux'. Write its SI unit.

(ii) What is the flux due to electric field $\vec{E} = 3 \times 10^3 \hat{i}$ N/C through a square of side 10 cm, when it is held normal to $\vec{E}$? *(AI 2015C)*

12. Given a uniform electric field $\vec{E} = 5 \times 10^3 \hat{i}$ N/C. Find the flux of this field through a square of 10 cm on a side whose plane is parallel to the y-z plane. What would be the flux through the same square if the plane makes a 30° angle with the x-axis? *(Delhi 2014)*

SA II (3 marks)

13. Consider a uniform electric field $\vec{E} = 3 \times 10^3 \hat{i}$ N/C. Calculate the flux of this field through a square surface of area 10 cm^2 when

(i) its plane is parallel to the y-z plane

(ii) the normal to its plane makes a 60° angle with the x-axis. *(Delhi 2013C)*

Electric Dipole

VSA (1 mark)

14. Define the term electric dipole moment of a dipole. State its S.I. unit. *(Foreign 2013, AI 2011)*

LA (5 marks)

15. An electric dipole of dipole moment $\vec{p}$ consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit $x >> a$, $\vec{E} \longrightarrow 2\vec{p}/(4\pi\varepsilon_0 x^3)$. *(3/5, Delhi 2015)*

16. Find the resultant electric field due to an electric dipole of dipole moment $2aq$ ($2a$ being the separation between the charges $\pm q$) at a point distance x on its equator. *(2/5, Foreign 2015)*

17. Define electric dipole moment. Is it a scalar or a vector quantity? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole. *(3/5, AI 2013)*

Dipole in a Uniform External Field

VSA (1 mark)

18. Write the expression for the torque $\vec{\tau}$ acting on a dipole of dipole moment $\vec{p}$ placed in an electric field $\vec{E}$. *(Foreign 2015)*

19. At what position is the electric dipole in uniform electric field in its most stable equilibrium position? *(AI 2008)*

SA II (3 marks)

20. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Obtain the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. *(Delhi 2015C)*

21. An electric dipole is kept in a uniform electric field. Derive an expression for the net torque acting on it and write its direction. State the conditions under which the dipole is in (i) stable equilibrium and (ii) unstable equilibrium. *(Delhi 2012C)*

22. Derive an expression for the torque experienced by an electric dipole kept in a uniform electric field. *(Delhi 2008)*

LA (5 marks)

23. (a) Define torque acting on a dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Express it in the vector form and point out the direction along which it acts.

(b) What happens if the field is non-uniform?

(c) What would happen if the external field $\vec{E}$ is increasing (i) parallel to $\vec{p}$ and (ii) anti-parallel to $\vec{p}$? *(Foreign 2016)*

24. Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$. *(3/5, AI 2014)*

Continuous Charge Distribution

SA I (2 marks)

25. Deduce the expression for the electric field $\vec{E}$ due to a system of two charges q_1 and q_2 with position vectors $\vec{r}_1$ and $\vec{r}_2$ at a point $\vec{r}$ with respect to the common origin O. *(Delhi 2010C)*

Gauss's Law

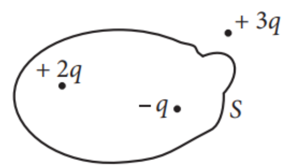
VSA (1 mark)

26. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased? *(Delhi 2016)*

27. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? *(Delhi 2015)*

28. A charge 'q' is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube? *(AI 2012)*

29. Figure shows three point charges, $+2q$, $-q$, $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface 'S'. What is the electric flux due to this configuration through the surface 'S'? *(Delhi 2010)*



30. A charge Q μC is placed at the centre of a cube. What is the electric flux coming out from any one surface? *(AI 2010)*

31. If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change? *(AI 2008)*

SA I (2 marks)

32. Show that the electric field at the surface of a charged conductor is given by $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction. *(AI 2010)*

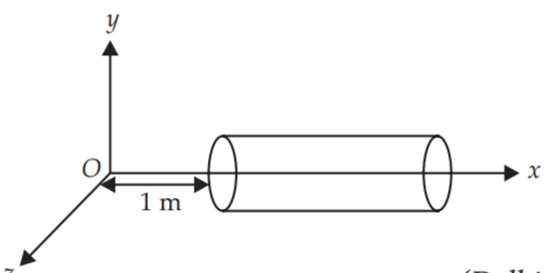
33. Define electric flux. Write its S.I. unit. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change? *(AI 2009)*

34. A sphere S_1 of radius r_1 encloses a charge Q, if there is another concentric sphere S_2 of radius $r_2 (r_2 > r_1)$ and there are no additional charges between S_1 and S_2. Find the ratio of electric flux through S_1 and S_2. *(AI 2009)*

35. Define electric flux. Write its S.I. unit. A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up and increases in size, how does the total electric flux coming out of the surface change? Give reason. *(Delhi 2007)*

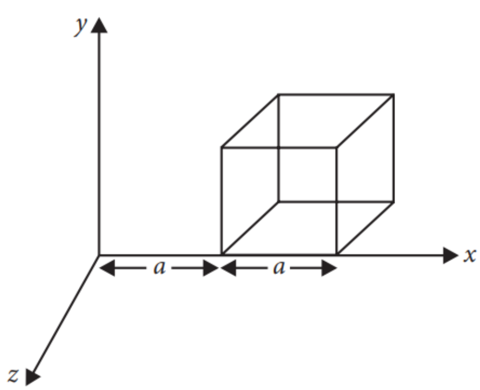
SA II (3 marks)

36. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50x\hat{i}$, where E is in N C^{-1} and x is in metres. Find
(i) net flux through the cylinder.
(ii) charge enclosed by the cylinder.



(Delhi 2013)

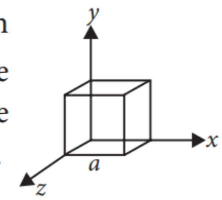
37. State Gauss's law in electrostatic. A cube with each side 'a' is kept in an electric field given by $\vec{E} = Cx\hat{i}$, (as is shown in the figure) where C is a positive dimensional constant. Find out



(i) the electric flux through the cube
(ii) the net charge inside the cube.

LA (5 marks)

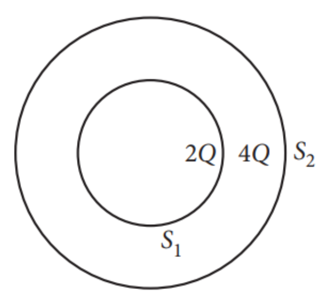
38. Given the electric field in the region $\vec{E} = 2x\hat{i}$, find the electric flux through the cube and the charge enclosed by it.



(2/5, Delhi 2015)

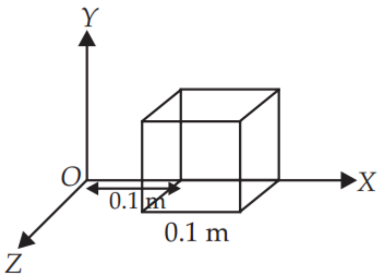
39. Define electric flux. Write its S.I. unit.
"Gauss's law in electrostatics is true for any closed surface, no matter what its shape or size is". Justify this statement with the help of a suitable example. *(AI 2015)*

40. Consider two hollow concentric spheres S_1 and S_2, enclosing charges $2Q$ and $4Q$ respectively as shown in figure.
(i) Find out the ratio of the electric flux through them.
(ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



(AI 2014)

41. (a) Define electric flux. Write its SI units.
(b) The electric field components due to a charge inside the cube of side 0.1m are as shown: $E_x = \alpha x$, where $\alpha = 500$ N/C-m
$E_y = 0, E_z = 0$.
Calculate (i) the flux through the cube, and (ii) the charge inside the cube.



(AI 2008)

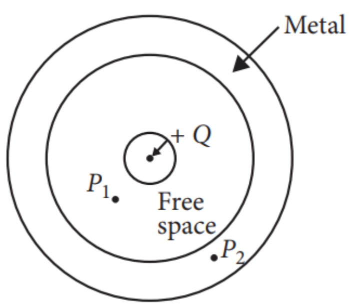
Applications of Gauss's Law

VSA (1 mark)

42. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius '$3a$' with its centre at the origin? *(AI 2013)*

SA I (2 marks)

43. A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2.



(AI 2014)

44. Two concentric metallic spherical shells of radii R and $2R$ are given charges Q_1 and Q_2 respectively. The surface charge densities on the outer surfaces of the shells are equal. Determine the ratio $Q_1 : Q_2$. *(Foreign 2013)*

45. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q. A charge q is placed at the centre of the shell.

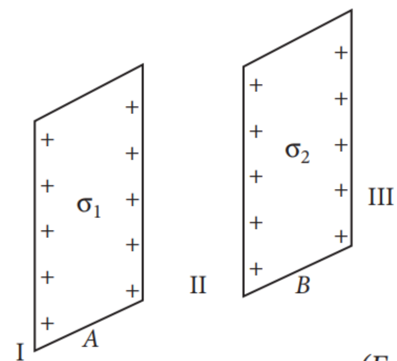
(a) What is the surface charge density on the (i) inner surface, (ii) outer surface of the shell?

(b) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell.

(AI 2010)

SA II (3 marks)

46. Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 $(\sigma_1 > \sigma_2)$ are shown in the figure. Write the magnitudes and directions of the net fields in the regions marked II and III.



(Foreign 2014)

47. (i) State Gauss's law.

(ii) A thin straight infinitely long conducting wire of linear charge density 'λ' is enclosed by a cylindrical surface of radius 'r' and length 'l'. Its axis coinciding with the length of the wire. Obtain the expression for the electric field, indicating its direction, at a point on the surface of the cylinder. *(Delhi 2012C)*

48. Using Gauss's law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for $r > R$ and $r < R$.

(Delhi 2011, 2009)

49. State Gauss's law in electrostatics. Using this law derive an expression for the electric field due to a uniformly charged infinite plane sheet. *(Delhi 2009)*

50. State Gauss's law in electrostatics. Use this law to derive an expression for the electric field due to an infinitely long straight wire of linear charge density λ Cm^{-1}. *(Delhi 2009)*

51. A positive point charge ($+q$) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate. Derive the expression for the electric field at the surface of a charged conductor. *(AI 2009)*

52. Use Gauss's law to derive the expression for the electric field between two uniformly charged large parallel sheets with surface charge densities $+\sigma$ and $-\sigma$ respectively. *(AI 2009)*

LA (5 marks)

53. Use Gauss's law to find the electric field due to a uniformly charged infinite plane sheet. What is the direction of field for positive and negative charge densities? *(AI 2016)*

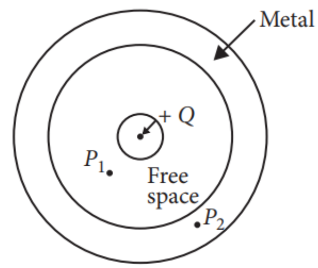
54. Use Gauss's law to prove that the electric field inside a uniformly charged spherical shell is zero. *(AI 2015)*

55. (a) A small conducting sphere of radius 'r' carrying a charge $+q$ is surrounded by a large concentric conducting shell of radius R on which a charge $+Q$ is placed. Using Gauss's law derive the expressions for the electric field at a point 'x'
(i) between the sphere and the shell ($r < x < R$).
(ii) outside the spherical shell.

56. Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside and (ii) inside the shell. Plot a graph showing variation of electric field as a function of $r > R$ and $r < R$. (r being the distance from the centre of the shell). *(AI 2013)*

57. Using Gauss's law, derive the expression for the electric field at a point (i) outside and (ii) inside a uniformly charged thin spherical shell. Draw a graph showing electric field $\vec{E}$ as a function of distance from the centre. *(AI 2013C)*

58. (i) Define electric flux. Write its S.I. unit.
(ii) A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2.



(iii) Draw the pattern of electric field lines in this arrangement. *(AI 2012C)*

59. (a) Using Gauss's law, derive an expression for the electric field intensity at any point outside a uniformly charged thin spherical shell of radius R and charge density σ C/m^2. Draw the field lines when the charge density of the sphere is (i) positive, (ii) negative.
(b) A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 $\mu C/m^2$. Calculate the
(i) charge on the sphere.
(ii) total electric flux passing through the sphere. *(Delhi 2008)*